Production of Jet Pairs at Large Relative Rapidity in Hadron-Hadron Collisions as a Probe of the Perturbative Pomeron

W.J. Stirling

Departments of Mathematical Sciences and Physics, University of Durham
Durham DH1 3LE, England

Abstract

The production of jet pairs with small transverse momentum and large relative rapidity in high energy hadron-hadron collisions is studied. The rise of the parton-level cross section with increasing rapidity gap is a fundamental prediction of the BFKL 'perturbative pomeron' equation of Quantum Chromodynamics. However, at fixed collider energy it is difficult to disentangle this effect from variations in the cross section due to the parton distributions. It is proposed to study instead the distribution in the azimuthal angle difference of the jets as a function of the rapidity gap. The flattening of this distribution with increasing dijet rapidity gap is shown to be a characteristic feature of the BFKL behaviour. Predictions for the Fermilab $p\bar{p}$ collider are presented.

1 Introduction

There is currently much interest in the QCD 'perturbative pomeron'. This is the phenomenon, obtained by resumming a certain type of soft gluon emission to all orders in the leading logarithm approximation using the Balitsky-Kuraev-Fadin-Lipatov (BFKL) equation [1], which is supposed to produce a sharp rise at small x in deep inelastic structure functions [2]. Recent measurements at HERA [3] may well show the first evidence for this behaviour, although it is premature to draw any definitive conclusions.

One of the difficulties with extracting information on the perturbative pomeron from structure function measurements alone is that both perturbative and non-perturbative effects are very likely intertwined in a non-trivial way, and therefore obtaining information on the former requires some sort of model-dependent subtraction of the latter. For this reason, attempts have been made to find other quantities which probe more directly the perturbative behaviour. In the context of deep inelastic scattering, one can look for an associated jet of longitudinal momentum fraction $x' \gg x_{\mathrm{Bj}}$ and comparable transverse momentum to the virtual photon momentum Q [4]. The BFKL behaviour is then reflected in the growth of the cross section with $\log(x'/x_{\mathrm{Bj}})$. Alternatively, Mueller and Navelet have shown [5] that in high-energy hadron-hadron collisions one can utilize a two-jet inclusive cross section where the jets have small transverse momentum and a large relative rapidity, Δy. The rise in the cross section with increasing Δy is then controlled by the perturbative pomeron.

It is this latter process that we investigate here. We first define the cross section introduced in Ref. [5], and show why it is difficult to measure under present experimental conditions. We then consider an alternative but closely-related quantity, the azimuthal angle correlation between jets at large relative rapidity, which we argue is more promising from an experimental point of view. Predictions are presented for the Fermilab $p\bar{p}$ collider at $\sqrt{s} = 1.8$ TeV. Our results also apply to the corresponding two-jet cross section in deep inelastic scattering as measured at HERA, although we shall not pursue this issue here.

Consider the inclusive two-jet cross section in $p\bar{p}$ (or pp) collisions at energy $\sqrt{s}$, where each jet has a minimum transverse momentum $M \ll \sqrt{s}$, and the jets are produced with equal and opposite large rapidity $\pm\frac{1}{2}\Delta$.[1] Adapting the results of Ref. [5], the cross section for this can be written in the form

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}y_1\mathrm{d}y_2}\right|_{y_1=-y_2=\frac{1}{2}\Delta} \simeq x_1 G(x_1, M^2)\, x_2 G(x_2, M^2)\, \hat{\sigma}(\alpha_s(M^2), M^2, \Delta) \qquad (1)$$

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[1]The restriction to *equal* and opposite rapidities is not crucial and is made here only to simplify the discussion.

where

$$G(x, \mu^2) \;=\; g(x, \mu^2) \;+\; \frac{4}{9} \sum_q (q(x, \mu^2) + \bar{q}(x, \mu^2)) \,,$$

$$x_1 \;=\; x_2 \;=\; \frac{2M}{\sqrt{s}} \cosh(\tfrac{1}{2}\Delta) \;\simeq\; \frac{M}{\sqrt{s}} e^{\Delta/2} \,,$$

$$\hat{\sigma}(\alpha_s, M^2, \Delta) \;=\; \left(\frac{\alpha_s C_A}{\pi}\right)^2 \frac{\pi^3}{2M^2} \left[1 + \sum_{n \geq 1} a_n \, (\alpha_s \Delta)^n + \dots \right] \,. \qquad (2)$$

Note the use of the 'effective subprocess approximation', appropriate here because of the dominance of small momentum transfer in the subprocess $\hat{t}$ channel. The ... in (2) refers to corrections outside the leading logarithm approximation implicit in (1), i.e. terms of order $\alpha_s^n \Delta^{n-1}$, $\alpha_s^n \Delta^{n-2}$, ... and power-correction terms suppressed by powers of $e^{-\Delta}$.

According to the BFKL 'perturbative pomeron' analysis, the subprocess cross section $M^2 \hat{\sigma}$ is expected to rise at large Δ. The asymptotic behaviour, ignoring possible 'parton saturation' effects [5], is predicted to be

$$M^2 \hat{\sigma} \sim e^{\lambda \Delta}, \quad \text{as } \Delta \to \infty \,, \qquad (3)$$

with λ a number of order 0.5. This is the analogue of the expected $x^{-\lambda}$ growth of the F_2 structure function at small x [2].

Now with $\sqrt{s} = 1.8$ TeV and $M \sim 10$ GeV, the maximum value of Δ is of order 10, which might at first sight appear sufficiently large to test the predicted BFKL behaviour.[2] The problem, however, is the additional Δ dependence in the cross section induced by the x dependence of the parton distributions in (1). At large Δ, the behaviour of the cross section will be completely dominated by the $x \to 1$ suppression of the parton distributions. This can only be avoided by increasing the energy $\sqrt{s}$ of the collider as Δ is increased, in such a way that x_1 and x_2 remain fixed, which is not an easy proposition in practice.

There are at least two ways around this difficulty. First, one could argue that the parton distributions at medium-to-large x are now sufficiently well known that they can be factored out of the measured cross section with sufficient accuracy. While this might be true for the quark component of $G(x, \mu^2)$, the gluon component is much less well known at large x. Even if it was, there is still a problem with scale dependence — the cross section in (1) is not known beyond leading logarithm accuracy, and so the choice of scale in the parton distributions is somewhat arbitrary.

A second possibility is to use the distribution in the azimuthal angle difference between the two jets as a signature for BFKL behaviour. When Δ is small, the cross

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[2]In fact, the predicted asymptotic behaviour appears to set in at relatively modest values of Δ, see Section 2 and Ref. [5].

section is dominated by the lowest order $2 \to 2$ scattering subprocesses and the jets are back-to-back in the transverse plane. As Δ increases, more and more soft gluons with transverse momentum $\sim M$ are emitted in the rapidity interval between the two fast jets, and the azimuthal correlation is gradually lost until, asymptotically, there is no correlation at all. The change in the overall weighting, due to variations in the parton distributions as Δ increases at fixed $\sqrt{s}$, has no effect on the shape of the azimuthal distribution.

The study of the azimuthal correlation between the two jets at large rapidity is the subject of the present analysis. We first of all derive the basic formulae, which is an extension of the treatment in [5], and then make numerical predictions for experimentally measurable quantities. We shall demonstrate that the weakening of the correlation should already be observable at the Fermilab $p\bar{p}$ collider.

2 BFKL formalism for dijet production

We start by considering the subprocess cross section for inclusive two-jet (i.e. parton) production. We are interested in jets produced with equal and opposite large longitudinal energy, $k_L \sim E$ where E is the parton beam energy in the parton-parton centre-of-mass, and small transverse momentum $k_T > M$ where $M \ll E$ is a fixed cut-off. The rapidity gap between the jets is then $\Delta y \equiv \Delta \simeq 2\log(E/M) \gg 1$. In practice, M will be a number of order 10 GeV and the maximum value of E is set by the kinematic limit $\sqrt{s}/2$.

At the Fermilab collider, a large fraction of such jet pairs are produced in gluon-gluon collisions, and so in what follows we focus on the subprocess $gg \to gg + X$. The quark initiated processes are reinstated afterwards using the effective subprocess approximation. If we assume, to begin with, that we can ignore the running of the strong coupling and take $\alpha_s = \alpha_s(M^2)$ as fixed, then the subprocess cross section of Eq. (1) can be written as [5]

$$\hat{\sigma}(\alpha_s, M^2, \Delta) = \left(\frac{\alpha_s C_A}{\pi}\right)^2 \frac{\pi^3}{2M^2} \int_{-\pi}^{\pi} \mathrm{d}\phi \, F(\phi, \Delta) \; . \tag{4}$$

Here we have introduced the variable $\phi = \pi - \phi_{jj}$ where ϕ_{jj} is the azimuthal angle difference between the two jets. Thus $\phi = 0$ corresponds to back-to-back jets in the transverse plane. In what follows we will be particularly interested in the differential distribution $\mathrm{d}\hat{\sigma}/\mathrm{d}\phi$, which is proportional to F.

The function F in Eq. (4) is simply the quantity $f(k_{T1}, k_{T2}, \Delta)$ defined in Ref. [5], integrated over $M^2 < k_{Ti}^2 < \infty$ at fixed ϕ. The latter function satisfies the BFKL equation (see Appendix). The solution for F is

$$F(\phi, \Delta) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} e^{in\phi} \, C_n(t) \; , \quad t = \frac{\alpha_s C_A}{\pi} \Delta \; , \tag{5}$$

3

where

$$C_n(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\mathrm{d}z}{z^2 + \frac{1}{4}} \, e^{2t\chi_n(z)},$$

$$\chi_n(z) = \mathrm{Re}\left[\psi(1) - \psi(\tfrac{1}{2}(1+|n|)+iz)\right], \tag{6}$$

and $\psi(x)$ is the logarithmic derivative of the gamma function. Note that this result corresponds to a perturbative expansion in powers of $\alpha_s \Delta \sim \alpha_s \log(E^2/M^2)$. It is these leading logarithms which have been resummed by the BFKL equation. Substituting back in the original expression for the cross section gives

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}y_1 \mathrm{d}y_2 \mathrm{d}\phi}\right|_{y_1=-y_2=\frac{1}{2}\Delta} \simeq x_1 G(x_1, M^2)\, x_2 G(x_2, M^2) \left(\frac{\alpha_s C_A}{\pi}\right)^2 \frac{\pi^3}{2M^2}\, F(\phi, \Delta), \tag{7}$$

with $x_1 = x_2 = 2M \cosh(\frac{1}{2}\Delta)/\sqrt{s}$. Before performing a full numerical calculation of this cross section, we discuss several important analytic results which follow from Eqs. (5,6).

2.1   Comparison with exact lowest-order calculation

Application of the BFKL formalism only makes sense in a kinematic region where the exact leading order $(2 \to 2)$ cross section is well-approximated by the first term in the perturbation series on the right-hand side of Eq. (7). The latter is obtained by setting $t = 0$ in Eq. (6), which gives

$$C_n(0) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\mathrm{d}z}{z^2 + \frac{1}{4}} = 1$$

$$\Rightarrow \quad F(\phi, \Delta) = \delta(\phi). \tag{8}$$

When this is substituted in Eq. (4), we obtain

$$M^2 \; \hat{\sigma}|_{\mathrm{LO}} = \tfrac{1}{2}\pi \alpha_s^2 C_A^2. \tag{9}$$

Note that in the $\Delta \to \infty$ limit the subprocess cross section scales as $1/M^2$. In this section we rederive this result starting from the exact $(2 \to 2)$ lowest order cross section, which will enable us to assess how rapidly the asymptotic behaviour is approached.

The two-jet inclusive cross section in leading order is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_1 \mathrm{d}y_2 \mathrm{d}p_T^2} = \frac{1}{16\pi^2 s} \sum_{a,b,c,d=q,g} x_1^{-1} f_a(x_1, \mu^2)\, x_2^{-1} f_b(x_2, \mu^2)\, \overline{\sum} |\mathcal{M}(ab \to cd)|^2, \tag{10}$$

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where $\overline{\sum}$ denotes the appropriate sums and averages over colours and spins. To begin with, we restrict our attention to the $gg \rightarrow gg$ subprocess. Setting $y_1 = -y_2 = \frac{1}{2}\Delta$ gives

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_1\mathrm{d}y_2\mathrm{d}p_T^2}\bigg|_{y_1=-y_2=\frac{1}{2}\Delta} = \frac{xg(x,\mu^2)\, xg(x,\mu^2)}{256\pi\, p_T^4\, \cosh^4(\frac{1}{2}\Delta)}\, \overline{\sum}\, |\mathcal{M}(gg \rightarrow gg)|^2 \,, \qquad (11)$$

where

$$x = \frac{2p_T}{\sqrt{s}}\, \cosh(\tfrac{1}{2}\Delta) \,, \qquad (12)$$

and the subprocess matrix element is evaluated at

$$\frac{\hat{t}}{\hat{s}} = -\frac{1}{2}\left(1 - \tanh(\tfrac{1}{2}\Delta)\right) \,, \qquad \frac{\hat{u}}{\hat{s}} = -\frac{1}{2}\left(1 + \tanh(\tfrac{1}{2}\Delta)\right) \,;\, . \qquad (13)$$

Integrating over the jet transverse momentum $p_T > M$ then gives

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_1\mathrm{d}y_2}\bigg|_{y_1=-y_2=\frac{1}{2}\Delta} = \frac{\overline{\sum}\, |\mathcal{M}(gg \rightarrow gg)|^2}{256\pi\, \cosh^4(\frac{1}{2}\Delta)}\, \int_{M^2}^{p_T^2(\mathrm{max})} \frac{\mathrm{d}p_T^2}{p_T^4}\, [xg(x,\mu^2)]^2 \,, \qquad (14)$$

with $p_T^2(\mathrm{max}) = \frac{1}{4}s\cosh^{-2}(\frac{1}{2}\Delta)$. By extracting a factor of M^{-2}, and ignoring logarithmic scaling violations in the parton distributions, the integral on the right-hand side becomes a function of the dimensionless quantity $X = 2M\cosh(\frac{1}{2}\Delta)/\sqrt{s}$.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_1\mathrm{d}y_2}\bigg|_{y_1=-y_2=\frac{1}{2}\Delta} = \frac{\overline{\sum}\, |\mathcal{M}(gg \rightarrow gg)|^2}{256\pi\, \cosh^4(\frac{1}{2}\Delta)}\, \frac{1}{M^2}\, \int_1^{X^{-2}} \frac{\mathrm{d}u^2}{u^4}\, \left[xg(x,\mu^2)\right]^2\bigg|_{x=Xu} \,. \qquad (15)$$

Next, consider the limit

$$1 \ll \Delta \ll \log\left(\frac{\sqrt{s}}{M}\right) \,. \qquad (16)$$

The first term on the right-hand side of Eq. (15) tends to a finite value of

$$\frac{\overline{\sum}\, |\mathcal{M}(gg \rightarrow gg)|^2}{256\pi\, \cosh^4(\frac{1}{2}\Delta)} \rightarrow \tfrac{1}{2}\pi C_A^2\alpha_s^2 \,. \qquad (17)$$

The integral over the parton distributions is dominated by the contribution from the lower limit, i.e.

$$\int_1^{X^{-2}} \frac{\mathrm{d}u^2}{u^4}\, \left[xg(x,\mu^2)\right]^2\bigg|_{x=Xu} \rightarrow \left[Xg(X,\mu^2)\right]^2 \qquad (18)$$

Combining the results from Eqs. (17,18) gives

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_1\mathrm{d}y_2}\bigg|_{y_1=-y_2=\frac{1}{2}\Delta} \simeq \tfrac{1}{2}\pi C_A^2\alpha_s^2\, \frac{1}{M^2}\, \left[Xg(X,\mu^2)\right]^2 \qquad (19)$$

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in agreement with the leading-order contribution in Eq. (7).

The next step is to investigate quantitatively how rapidly the asymptotic result is attained in practice. We focus first on the matrix element part, Eq. (17). Figure 1 shows the ratio of the left-hand side (the exact result) to the right-hand side (the asymptotic result), as a function of the rapidity gap Δ. Evidently, the two agree to better than 10% for $\Delta > 3.3$. This is not unexpected, since we are testing here the size of the 'power correction' terms of order $e^{-\Delta}$. We can also test the effective subprocess approximation, by comparing the $qg \rightarrow qg$ and $q\bar{q} \rightarrow q\bar{q}$ amplitudes, scaled by $9/4$ and $(9/4)^2$ respectively, to the asymptotic $gg \rightarrow gg$ result. These ratios are shown as the dashed (qg) and dash-dotted ($q\bar{q}$) lines in Fig. 1. The approach to the limiting form is very similar to the gg amplitude, indicating that in the large Δ region the effective subprocess approximation is valid.

We have already discussed how when M and $\sqrt{s}$ are fixed the bulk of the Δ dependence comes from the parton distributions. This is illustrated in Fig. 2, where the cross section of Eq. (15), scaled by M^2, is shown as a function of Δ. We have chosen $\sqrt{s} = 1.8\,\text{TeV}$, and used the latest 'MRS(H)' partons [8] with $\Lambda_{\overline{\text{MS}}}^{(4)} = 230\,\text{MeV}$. The scales in the running coupling and in the parton distributions are both set equal to M. The solid curves correspond to the exact $gg \rightarrow gg$ cross section, for the representative values $M = 10\,\text{GeV}$ and $M = 30\,\text{GeV}$. Also shown (dashed lines) are the asymptotic cross sections defined by Eq. (19). There is a broad range of Δ where the shape of the exact cross sections is reasonably well approximated. The normalization, however, is too high by a factor of order two.[3] This can be traced to the fact that Eq. (18) is only valid when both X is small *and* the function $xg(x, \mu)$ is slowly varying. If $xg \sim x^{-\lambda}$ in the relevant x region, then an error of order $(1 + \lambda)$ is made in the normalization.

2.2 Comparison with exact $O(\alpha_s^3)$ calculation for $\phi \neq 0$

Expanding the exponential inside the integral in Eq. (6) allows us to read off the differential ϕ-distribution at next-to-lowest order, i.e. $O(\alpha_s^3)$:

$$M^2 \left.\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\phi}\right|_{\text{NLO}} = \left(\frac{\alpha_s C_A}{\pi}\right)^3 \frac{\pi^3}{2} \Delta\, f(\phi)\,, \tag{20}$$

where

$$f(\phi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} e^{in\phi}\, c_n\,, \tag{21}$$

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[3]In the range $2 < \Delta < 6$ for $M = 10$ GeV the ratio of the exact to the approximate cross section always lies in the range 0.45 to 0.55.

and the Fourier coefficients are

$$c_n = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{dz}{z^2 + \frac{1}{4}} \chi_n(z) = 2\left[\psi(1) - \psi(1 + \tfrac{1}{2}|n|)\right]. \tag{22}$$

The coefficients can be calculated explicitly by contour integration in the complex z plane:

$$c_n = c_{-n}, \qquad c_{n+2} = c_n - \frac{4}{2+n} \quad (n \geq 0),$$
$$c_0 = 0, \qquad c_1 = 4\left(\log 2 - 1\right). \tag{23}$$

The first point to note is that since $c_0 = 0$, the integral of $f(\phi)$ vanishes, i.e.

$$M^2 \; \hat{\sigma}|_{\mathrm{NLO}} = 0. \tag{24}$$

To calculate the ϕ distribution we have to substitute the c_n coefficients into Eq. (21) and sum the Fourier series. We have found it easier, however, to start from the solution to the BFKL equation in transverse momentum space, where for $\phi \neq 0$ (see Appendix),

$$
\begin{aligned}
f(\phi) &= \frac{M^2}{2\pi} \int_{M^2}^{\infty} \int_{M^2}^{\infty} \frac{dk_{T1}^2 \, dk_{T2}^2}{k_{T1}^2 k_{T2}^2 (k_{T1}^2 + k_{T2}^2 - 2k_{T1}k_{T2}\cos\phi)} \\
&= \frac{1}{\pi} \left[\log(2(1 - \cos\phi)) + (\pi - \phi)\cot\phi\right] \quad \text{for } 0 < \phi \leq \pi, \tag{25}
\end{aligned}
$$

and $f(-\phi) = f(\phi)$. The singular behaviour $f \sim \phi^{-1}$ as $\phi \to 0$, which corresponds to almost back-to-back jets, arises from soft emission of the third (gluon) jet [1]. It is cancelled in the total cross section by a virtual gluon contribution proportional to $\delta(\phi)$. This can be taken into account by invoking the standard 'plus prescription', i.e. $f(\phi) \to [f(\phi)]_+$ where

$$\int_{-\pi}^{\pi} d\phi \, g(\phi) \, [f(\phi)]_+ = \int_{-\pi}^{\pi} d\phi \, (g(\phi) - g(0)) \, f(\phi). \tag{26}$$

It is important to note that the distribution we have derived is a leading logarithm result, in that it corresponds to retaining only the leading $\alpha_s \Delta$ contribution and ignoring corrections of order α_s, $\alpha_s e^{-\Delta}$ etc. To study the validity of this approximation, we can compare the result in Eq. (25) with an exact calculation based on the complete $gg \to ggg$ matrix element. The analogue of the $2 \to 2$ cross section Eq. (14) is

$$
\begin{aligned}
\left.\frac{d\sigma}{dy_1 dy_2 d\phi}\right|_{y_1 = -y_2 = \frac{1}{2}\Delta} &= \frac{1}{512\pi^4} \int_{M^2} dk_{T1}^2 \int_{M^2} dk_{T2}^2 \int_{-\frac{1}{2}\Delta}^{\frac{1}{2}\Delta} dy_3 \\
&\quad \times \left[x_1 g(x_1, \mu^2) \, x_2 g(x_2, \mu^2)\right] \\
&\quad \times \hat{s}^{-2} \overline{\sum} |\mathcal{M}(gg \to ggg)|^2 \tag{27}
\end{aligned}
$$

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where

$$
\begin{aligned}
k_{T3}^2 &= k_{T1}^2 + k_{T2}^2 - 2k_{T1}k_{T2}\cos\phi \\
x_1 &= (k_{T1}e^{\frac{1}{2}\Delta} + k_{T2}e^{-\frac{1}{2}\Delta} + k_{T3}e^{y_3})/\sqrt{s} \\
x_2 &= (k_{T1}e^{-\frac{1}{2}\Delta} + k_{T2}e^{\frac{1}{2}\Delta} + k_{T3}e^{-y_3})/\sqrt{s} \\
\hat{s} &= k_{T1}^2 + k_{T2}^2 + k_{T3}^2 + 2k_{T1}k_{T2}\cosh\Delta \\
&\quad + 2k_{T1}k_{T3}\cosh(\tfrac{1}{2}\Delta - y_3) + 2k_{T2}k_{T3}\cosh(\tfrac{1}{2}\Delta + y_3) \; .
\end{aligned}
\tag{28}
$$

According to the BFKL analysis, this cross section should have the asymptotic limit

$$
\left.\frac{\mathrm{d}\sigma}{\mathrm{d}y_1\mathrm{d}y_2\mathrm{d}\phi}\right|_{y_1=-y_2=\frac{1}{2}\Delta} = \left(\frac{\alpha_s C_A}{\pi}\right)^3 \frac{\pi^3}{2M^2}\,\Delta\,f(\phi)\,[Xg(X,\mu^2)]^2 \; .
\tag{29}
$$

We can see how this behaviour arises: the matrix element in (27) is dominated by configurations where the third gluon jet is produced centrally, i.e. $|y_3| \ll \frac{1}{2}\Delta$. With the matrix element approximated by its value at $y_3 = 0$, the y_3 integral gives the overall factor of Δ, and the remaining k_{Ti} integrals give the function $f(\phi)$, as in Eq. (25). The parton distributions are again dominated by their values at $x_1 = x_2 = X = 2M\cosh(\frac{1}{2}\Delta)/\sqrt{s}$, as for the leading order cross section.

We can study the approach to the asymptotic result in two stages. First, at the subprocess level, we can compare the cross section

$$
\left.\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y_1\mathrm{d}y_2\mathrm{d}\phi}\right|_{y_1=-y_2=\frac{1}{2}\Delta} = \frac{1}{512\pi^4}\int_{M^2}^{\infty}\mathrm{d}k_{T1}^2\,\mathrm{d}k_{T2}^2\int_{-\frac{1}{2}\Delta}^{\frac{1}{2}\Delta}\mathrm{d}y_3\,\hat{s}^{-2}\overline{\sum}|\mathcal{M}(gg\to ggg)|^2
\tag{30}
$$

with the asymptotic form

$$
\left.\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y_1\mathrm{d}y_2\mathrm{d}\phi}\right|_{y_1=-y_2=\frac{1}{2}\Delta} = \left(\frac{\alpha_s C_A}{\pi}\right)^3 \frac{\pi^3}{2M^2}\,\Delta\,f(\phi) \; .
\tag{31}
$$

Figure 3 compares the ϕ distribution calculated from Eq. (30) with the function $f(\phi)$, for $\Delta = 4, 8, 12$. The exact calculation has been scaled by the same factors multiplying $f(\phi)$ on the right-hand side of Eq. (31), so that the exact and approximate distributions should coincide in the limit $\Delta \to \infty$. The results confirm the approach to the asymptotic distribution. At small ϕ, the asymptotic behaviour is already a good approximation for $\Delta = 4$, while the convergence is slower at large ϕ. This is presumably because at large ϕ the third gluon can have significant transverse momentum and energy, thus invalidating the approximations under which the BFKL equation is derived and leading to sizeable sub-asymptotic corrections.

Figure 4 makes the same comparison at the cross-section level, for $p\bar{p}$ collisions with $\sqrt{s} = 1.8$ TeV, $M = 10$ GeV and $\Delta = 4, 6, 8$. The curves correspond to the exact and asymptotic cross sections of Eqs. (27) and (29) respectively. The constraints $x_1, x_2 \leq 1$ now give upper limits on the transverse momentum integrals. As for the leading-order case, the *normalization* is overestimated by the asymptotic form, but evidently the *shape* of the ϕ distribution is reasonably well approximated even at moderate Δ. The rapid change in the distribution with increasing Δ supports our assertion that the azimuthal distribution of the jets should be a more reliable indicator of BFKL behaviour than the overall ϕ-integrated cross section.

2.3 All-orders ϕ distribution

Through next-to-lowest order, then, we have

$$
\begin{aligned}
M^2 \frac{\mathrm{d}\hat{\sigma}(E, M)}{\mathrm{d}\phi} &= \left(\frac{\alpha_s C_A}{\pi}\right)^2 \frac{\pi^3}{2} \, F(\phi, \Delta) \\
F(\phi, \Delta) &= \delta(\phi) + \left(\frac{\alpha_s C_A}{\pi}\right) \Delta \, [f(\phi)]_+ + \dots ,
\end{aligned}
\tag{32}
$$

where the ... represent terms $O((\alpha_s \Delta)^n)$ with $n \geq 2$. Formally, the first few terms in this series will be a good approximation to the all-orders distribution provided $\Delta \gg 1$ and $\alpha_s \Delta \ll 1$. As the second of these inequalities is relaxed, higher order terms become more and more important. The inclusion of all terms of the form $(\alpha_s \Delta)^n$, via Eqs. (5,6), requires a numerical calculation and will be discussed below. First, we consider the limit $\alpha_s \Delta \gg 1$, where an analytic approximation can again be obtained.

To calculate the distribution in the asymptotic limit $\alpha_s \Delta \to \infty$, we return to Eq. (6) and use a saddle-point method [1] to evaluate the Fourier coefficients in the large $t = \alpha_s C_A \Delta/\pi$ limit. We expand the $\chi_n(z)$ about the saddle point at $z = 0$,

$$
\chi_n(z) = a_n - b_n z^2 + \dots ,
\tag{33}
$$

where

$$
\begin{aligned}
a_0 = 2 \log 2, \; a_1 = 0, &\qquad a_{n+2} = a_n - \frac{2}{1+n}, \quad (n \geq 0) \\
b_0 = 7\zeta(3), \; b_1 = \zeta(3), &\qquad b_{n+2} = b_n - \frac{8}{(1+n)^3}, \quad (n \geq 0)
\end{aligned}
\tag{34}
$$

from which the asymptotic $t \to \infty$ behaviour follows,

$$
C_n(t) \sim \frac{1}{\sqrt{\frac{1}{2}\pi b_n t}} \, e^{2 a_n t} .
\tag{35}
$$

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Figure 5 shows the first seven C_n coefficients as functions of t. Note that because $a_n < 0$ for $n \geq 1$, all but the C_0 coefficient tend to zero as $t \to \infty$. The asymptotic ϕ distribution is then obtained by substitution in Eq. (5),

$$
F(\phi, \Delta) \sim \frac{1}{2\pi} \left[\frac{1}{\sqrt{\frac{1}{2}\pi 7\zeta(3)t}} e^{4\log 2 \, t} + \frac{2\cos\phi}{\sqrt{\frac{1}{2}\pi\zeta(3)t}} \right.
$$
$$
\left. + \frac{2\cos 2\phi}{\sqrt{\frac{1}{2}\pi(7\zeta(3)-8)t}} e^{4(\log 2 - 1) \, t} + \dots \right]. \tag{36}
$$

Asymptotically, then, the ϕ distribution becomes *flat*. The emission of an infinite number of soft gluons has completely smeared out the back-to-back correlation exhibited by the lowest contributions to the perturbation series. Note that we have also reproduced the asymptotic result for the ϕ-integrated cross section [1, 5]:

$$
M^2 \hat\sigma(\alpha_s, M^2, \Delta) = \left(\frac{\alpha_s C_A}{\pi}\right)^2 \frac{\pi^2}{4} C_0(t) \longrightarrow \left(\frac{\alpha_s C_A}{\pi}\right)^2 \frac{\pi^2}{4} \frac{1}{\sqrt{\frac{1}{2}\pi 7\zeta(3)t}} e^{4\log 2 \, t} . \tag{37}
$$

For large Δ, therefore, we obtain the result given in Eq. (3) with

$$
\lambda = \frac{\alpha_s}{\pi} 4 \, C_A \log 2 = 0.5 , \tag{38}
$$

for $\alpha_s = 0.19$. The ϕ-integrated cross section was studied in some detail in Ref. [5], where an analytic approximation valid for $t \lesssim 1$ was derived. Figure 6 shows the function $C_0(t)$, (i) computed exactly using Eq. (6) (solid line), (ii) according to the analytic approximation of Ref. [5] (dotted line), and (iii) in the asymptotic limit, Eq. (37) [1] (dashed line).

We have so far obtained analytic approximations for the small t and large t behaviours of the differential ϕ distribution. The distribution at arbitrary t requires a numerical calculation of the sum and integral in Eqs. (5,6). Thus, Fig. 7 shows the function $F(\phi, \Delta)$ defined in Eq. (5) for $t = \alpha_s C_A \Delta / \pi = 0.25, 0.5, 1.0, 1.5$. We see very clearly the transition from a sharply peaked distribution at small t — recall that $F = \delta(\phi)$ at $t = 0$ — to a larger, flatter distribution as t increases. As the rapidity gap widens, the emission of more and more soft gluons uniformly 'fills in' the distribution at large ϕ.

3 Predictions for $p\bar{p}$ collisions at 1.8 TeV

The most direct test of the BFKL perturbative pomeron behaviour is the rise in the subprocess cross section with increasing rapidity gap Δ, i.e. $M^2 \hat\sigma \sim \exp(\lambda\Delta)$.

However, as discussed in the Introduction, one cannot yet regard this as a precision prediction of the theory. One particular issue concerns the inclusion of a running coupling in the BFKL analysis, i.e. $\alpha_s \rightarrow \alpha_s(k_T^2)$. This prevents the integrals being extended down to $k_T = 0$, thereby inducing a weak dependence on an infra-red cut-off parameter, see for example Ref. [7]. Increasing this cut-off reduces the phase space for the soft gluon emission and weakens the growth in the cross section with Δ. In addition, the subleading logarithmic corrections to the BFKL result are not yet known. As we have stressed, at fixed hadron collider energy the BFKL behaviour is anyway masked by additional dependence on Δ coming from the parton distributions. To investigate this latter effect quantitatively, we show in Fig. 8 the cross section of Eq. (1) at $\sqrt{s} = 1.8$ TeV as a function of Δ for two choices of minimum jet transverse momentum, $M = 10$ GeV and $M = 30$ GeV. The dashed curves correspond to the leading order contribution to $\hat{\sigma}$, i.e. the first term on the right-hand side in Eq. (2), while the solid curves are the all-orders BFKL result, corresponding to $\hat{\sigma}$ given in Eq. (37). The parton distributions are the latest MRS(H) set [8], with $\Lambda_{\overline{\text{MS}}}^{(4)} = 230$ MeV, which are consistent with both the recent HERA and the fixed-target F_2 measurements. Evidently, the shapes of the lowest-order and all-orders distributions are quite different. Notice that the x-dependence of the parton distributions more than compensates the BFKL rise in the subprocess cross section, so that the net effect is a cross section which *decreases* as a function of Δ. However, we should recall from Section 2.1 that the lowest-order approximation is only a good representation of the *shape* of the exact lowest order $2 \rightarrow 2$ cross section for large $\Delta \gtrsim 4$. The signature for BFKL behaviour is therefore a slower fall-off of the cross section with increasing Δ than predicted by the leading order (exact or approximate) cross section. The size of the effect can be gauged from Fig. 8. Whether the difference is detectable in practice depends on the precision with which jets at large rapidity can be reconstructed and measured experimentally.

We turn next to the distribution in the azimuthal angle difference of the two jets. Figures 9 and 10 show the $\phi = \pi - \phi_{jj}$ distributions, at fixed Δ, for $\sqrt{s} = 1.8$ TeV with (a) $M = 10$ GeV and (b) $M = 30$ GeV. In Fig. 9 the cross section itself is shown, while in Fig. 10 the distributions are normalized to have unit area for each value of Δ. The trend is that as Δ increases, the cross sections get smaller and the distributions become flatter in ϕ. The higher the transverse momentum cutoff M, the faster the decrease with Δ and the slower the approach to the flat distribution. For $M = 10$ GeV, values of Δ up to about 8 appear to be accessible, at least in principle. For this rapidity gap, the ϕ distribution is almost flat. A simpler representation of the flattening behaviour is provided by the *average* of $\cos \phi$, which is proportional to the C_1 coefficient of Eq. (6), i.e.

$$\langle \cos \phi \rangle = \frac{C_1(t)}{C_0(t)}, \qquad t = \frac{\alpha_s(M^2) C_A \Delta}{\pi} \tag{39}$$

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Figure 11 shows this average as a function of Δ, for $M = 10$ GeV and $M = 30$ GeV. The approach to flatness ($\langle \cos \phi \rangle \to 0$) is slower for the higher cut-off because of the smaller coupling constant.

4   Conclusions

In this study we have focussed on two important aspects of BFKL perturbative pomeron behaviour. The first is the rise in the cross section as the rapidity gap between two moderate p_T jets increases, as first discussed in Ref. [5]. The difficulty with this signature is that in order to circumvent the additional dependence on rapidity induced by the parton distributions, it is necessary to scale up the collider energy as the rapidity gap increases. At fixed collider energy, the dominant effect at large rapidity gap is the suppression of the cross section by the fall-off in the parton distributions as $x \to 1$. After allowing for the effects of jet reconstruction and measurement in the detector, it is not clear whether the relatively small effects of the BFKL behaviour can be observed. This is not, however, as severe a problem for the second important feature of the BFKL pomeron — the weakening of the correlation in the azimuthal angle of the two jets. The distribution in the azimuthal angle difference ϕ changes from being back-to-back for central jet pairs with a small rapidity difference, to an asympototically flat distribution as the jets separate in rapidity.

We have presented predictions for the ϕ distribution at different rapidity gaps Δ in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV, based on the fixed-coupling solutions of the BFKL equation. However, there are very likely non-negligible sub-leading corrections to this behaviour. We have investigated some of these by comparing the leading-logarithm predictions with those based on the exact low-order matrix elements. From this comparison, it seems that the leading behaviour may already be dominant for rapidity gaps as small as 4.

Of course, we have *not* included such important effects as smearing of the jet energies and angles by the jet algorithms used in the actual experiments. A more precise analysis would require a correspondong smearing of the ϕ distribution, which would inevitably weaken the correlations implied by perturbation theory alone. The size of this smearing could be estimated, for example, by comparing the actual ϕ distribution of two central jets — which should be well-described by lowest order perturbation theory — with the naive $\delta(\phi)$ expectation. If the smearing was parametrizable by, say, a gaussian distribution, this could be folded in to the Fourier coefficients in the perturbative predictions. We have not performed such an analysis here, since the form of the smearing is presumably detector and jet algorithm dependent.

In summary, therefore, it would be interesting to measure the azimuthal angle distribution of the two-jet inclusive cross section as a function of the jet rapidity gap, to see if the data are at least qualitatively in line with the flattening of the distribution

predicted by the BFKL equation. In this study we have only considered the case of $p\bar{p}$ collisions at 1.8 TeV. However, basically the same behaviour should also be manifest in any high-energy collider with quarks and gluons in the initial state. In particular, photoproduction of jet pairs at HERA could also be a useful place to look for evidence of the BFKL behaviour, and, of course, high-energy proton-proton colliders such as the LHC will allow a much larger range of rapidities to be covered.

Appendix: Solution of the BFKL equation

In this Appendix, we present a brief derivation of the solution of the BFKL equation for the two-jet inclusive cross section. More details can be found in Refs. [1, 5].

We start from the subprocess cross section $gg \rightarrow gg + X$, where the two final state gluons are produced with transverse momenta k_{Ti} at large rapidity separation Δ, and X represents additional soft gluons. The differential cross section can be written, following the notation of [5],

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^2 k_{T1} \mathrm{d}^2 k_{T2}} = \alpha_s^2 C_A^2 \, \frac{f(k_{T1}, k_{T2}, \Delta)}{k_{T1}^2 k_{T2}^2} \; . \tag{A1}$$

The Laplace transform of the function f,

$$\tilde{f}(k_{T1}, k_{T2}, \omega) = \int_0^\infty \mathrm{d}\Delta \; e^{-\omega\Delta} \; f(k_{T1}, k_{T2}, \Delta) \; , \tag{A2}$$

satisfies the BFKL equation [1]

$$\omega \tilde{f}(k_{T1}, k_{T2}, \omega) = \delta(k_{T1}^2 - k_{T2}^2)\delta(\phi_1 - \phi_2) + \left(\frac{\alpha_s C_A}{\pi^2} \right)$$
$$\times \int \frac{\mathrm{d}^2 k_T}{(k_{T1} - k_T)^2} \left[\tilde{f}(k_T, k_{T2}, \omega) - \frac{k_{T1}^2 \tilde{f}(k_{T1}, k_{T2}, \omega)}{k_T^2 + (k_{T1} - k_T)^2} \right] . \tag{A3}$$

This equation can be solved in closed form by introducing the Fourier transform of $\tilde{f}$ with respect to $\phi_1 - \phi_2$ and $\log(k_{T1}^2/k_{T2}^2)$:

$$\tilde{f}(k_{T1}, k_{T2}, \omega) = \frac{1}{2\pi} \sum_n e^{in(\phi_1 - \phi_2)} \frac{1}{2\pi} \int_{-\infty}^\infty \mathrm{d}z \; e^{-iz \log(k_{T1}^2/k_{T2}^2)} \; \tilde{f}_n(z, \omega). \tag{A4}$$

Substituting this into Eq. (A3) gives

$$\omega \tilde{f}_n(z, \omega) = \left(k_{T1}^2 k_{T2}^2 \right)^{-\frac{1}{2}} + \omega_0(n, z) \, \tilde{f}_n(z, \omega) \; , \tag{A5}$$

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where

$$\omega_0(n, z) \;=\; \left(\frac{\alpha_s C_A}{\pi} \right) \, 2\chi_n(z) \; , \qquad (A6)$$

with the function χ_n given in Eq. (6). Performing the inverse transform of Eq. (A2) then gives

$$f(k_{T1}, k_{T2}, \Delta) = \frac{1}{2\pi} \sum_n e^{in\phi} \, \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}z \, (k_{T1}^2)^{-\frac{1}{2} - iz} (k_{T2}^2)^{-\frac{1}{2} + iz} \, e^{2t\chi_n(z)} \; , \qquad (A7)$$

where $\phi = \phi_1 - \phi_2$ and $t = \alpha_s C_A \Delta / \pi$. The final step is to integrate the transverse momenta over the range $M^2 < k_{Ti}^2 < \infty$,

$$
\begin{aligned}
\hat{\sigma}(\alpha_s, M^2, \Delta) \;&=\; \frac{\pi}{2} \int_{M^2}^{\infty} \mathrm{d}k_{T1}^2 \int_{M^2}^{\infty} \mathrm{d}k_{T2}^2 \int_{-\pi}^{\pi} \mathrm{d}\phi \, \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^2 k_{T1} \mathrm{d}^2 k_{T2}} \\
&=\; \frac{\alpha_s^2 C_A^2 \pi}{2M^2} \int_{-\pi}^{\pi} \mathrm{d}\phi \, \frac{1}{2\pi} \sum_n e^{in\phi} \, \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{z^2 + \frac{1}{4}} \, e^{2t\chi_n(z)} \; , \qquad (A8)
\end{aligned}
$$

which gives the subprocess cross section of Eqs. (4,5,6).

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Figure Captions

[1] Ratio of the exact $2 \to 2$ subprocess cross sections defined in the text (Eq. (17)) to the asymptotic $gg \to gg$ scaling form given in Eq. (9), as a function of the rapidity gap Δ. The curves are (i) $gg \to gg$ (solid line), (ii) $qg \to qg$ multiplied by $9/4$ (dashed line), and (iii) $q\bar{q} \to q\bar{q}$ multiplied by $(9/4)^2$ (dot-dashed line).

[2] The lowest order $2 \to 2$ cross section of Eq. (15) as a function of the rapidity gap Δ, at $\sqrt{s} = 1.8$ TeV. The MRS(H) parton distributions [8] with $\Lambda_{\overline{\text{MS}}}^{(4)} = 230$ MeV are used. The solid curves correspond to the exact $gg \to gg$ cross section with minimum jet transverse momenta $M = 10$ GeV and $M = 30$ GeV. Also shown (dashed lines) are the asymptotic cross sections defined by Eq. (19).

[3] The asymptotic azimuthal angle distribution $f(\phi)$ of Eq. (25), compared to the distribution calculated from the exact $gg \to ggg$ matrix element for $\Delta = 4, 8, 12$ (dashed lines).

[4] The dependence of the differential cross section at $O(\alpha_s^3)$ on the azimuthal angle difference of the two jets with rapidity gap $\Delta = 4, 6, 8$, in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV with $M = 10$ GeV. The MRS(H) parton distributions [8] with $\Lambda_{\overline{\text{MS}}}^{(4)} = 230$ MeV are used. The solid curves correspond to the exact cross section (Eq. (27)), and the dashed curves to the asymptotic approximation of Eq. (29).

[5] The Fourier coefficients C_n ($n \leq 6$) defined in Eq. (6) as functions of $t = \alpha_s C_A \Delta / \pi$.

[6] The Fourier coefficient $C_0(t)$, which gives the ϕ-integrated cross section, (i) computed exactly using Eq. (6) (solid line), (ii) according to the analytic approximation of Ref. [5] (dotted line), and (iii) in the asymptotic limit, Eq. (37) (dashed line).

[7] The azimuthal angular distribution function $F(\phi, \Delta)$, defined in Eq. (5), for $t = \alpha_s C_A \Delta / \pi = 0.25, 0.5, 1.0, 1.5$.

[8] The cross section $\mathrm{d}\sigma/\mathrm{d}y_1\mathrm{d}y_2(y_1 = -y_2 = \frac{1}{2}\Delta)$ of Eq. (1) at $\sqrt{s} = 1.8$ TeV as a function of Δ, for two choices of minimum jet transverse momentum, $M = 10$ GeV and $M = 30$ GeV. The dashed curves correspond to the leading order contribution to $\hat{\sigma}$, i.e. the first term on the right-hand side in Eq. (2), and the solid curves are the all-orders BFKL result, corresponding to $\hat{\sigma}$ given in Eq. (37). The MRS(H) parton distributions [8] with $\Lambda_{\overline{\mathrm{MS}}}^{(4)} = 230$ MeV are used.

[9] The $\phi = \pi - \phi_{jj}$ distributions, at fixed Δ, for $\sqrt{s} = 1.8$ TeV with (a) $M = 10$ GeV and (b) $M = 30$ GeV. The MRS(H) parton distributions [8] with $\Lambda_{\overline{\mathrm{MS}}}^{(4)} = 230$ MeV are used.

[10] As for Fig. 9, but with the distributions normalized to unit area at each Δ.

[11] The average azimuthal angle difference $\langle \cos\phi \rangle = -\langle \cos\phi_{jj} \rangle$ as a function of Δ, for $M = 10$ GeV and $M = 30$ GeV.

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